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TWO-DIMENSIONAL PROBLEM OF PERIODIC LOADING OF AN ELASTIC PLATE FLOATING ON THE SURFACE OF AN INFINITELY DEEP FLUID

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The action of external periodic pressure on an elastic plate floating on the surface of a fluid assumed to be ideal and incompressible is examined by the method of normal modes in the linear formulation. The behavior of the matrix of coefficients of the hydrodynamic load on the plate is considered in detail for different frequencies. The behavior of the plate under localized periodic loading is compared for the cases of a heavy fluid with a finite or infinite depth and for a weightless infinite-depth fluid.

Key words: *floating elastic plate, periodic pressure, hydrodynamic load.*

Introduction. The importance of studying the unsteady behavior of floating elastic plates under the action of external loading is motivated by creation of large-scale floating platforms. The simplest example of the unsteady action on the plate is a periodic external pressure. This problem was considered for a beam plate in the two-dimensional formulation and for a circular plate in the three-dimensional case [1, 2]. The depth of the fluid was assumed to be finite. The methods used (expansion in terms of vertical eigenmodes [1] and Wiener–Hopf method [2]) did not allow extension of the results obtained in the entire frequency range to the case of an infinite depth of the fluid.

The method of normal modes is used in the present paper, and the hydrodynamic load (added mass and damping coefficients) is determined for each mode of plate oscillations. The behavior of the hydrodynamic load in the limiting cases with high and low oscillation frequencies is examined. The results obtained are used to solve the problem of the action of an external periodic pressure on a floating plate.

1. Formulation of the Problem. Let a thin elastic plate of width $2L$ and infinite length be floating on the surface of an infinitely deep ideal incompressible fluid. The fluid surface not covered by the plate is free. The ends of the plate are not fixed, and the plate draft is ignored. The plate is subjected to a periodic (with a frequency ω) external pressure of the form $P(x)\exp(i\omega t)$ (x is the horizontal coordinate directed perpendicular to the plate ends; $x = 0$ corresponds to the plate centerline).

The problem under consideration is two-dimensional. The fluid flow is assumed to be potential, and the oscillations of the plate and the fluid are assumed to be periodic in time. The fluid-velocity potential $\phi(x, y, t)$ and the normal deflection of the plate $w(x, t)$ are sought in the form

$$\phi(x, y, t) = i\omega\Phi(x, y)\exp(i\omega t), \qquad w(x, t) = W(x)\exp(i\omega t),$$

where the y axis is directed vertically upward; $y = 0$ on the fluid surface.

Within the framework of the linear wave theory, the velocity potential satisfies the Laplace equation

$$\Delta\Phi = 0 \qquad (|x| \leqslant \infty, \quad y < 0)$$

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and the boundary conditions

$$\frac{\partial\Phi}{\partial y} = W(x) \quad (|x| \leqslant L,\ y = 0), \qquad \frac{\partial\Phi}{\partial y} = \nu\Phi \quad (|x| > L,\ y = 0), \qquad \nu = \omega^2/g,$$

$$|\nabla\Phi| \to 0 \qquad (y \to -\infty), \tag{1.1}$$

where g is the acceleration of gravity. The normal deflection of the plate is described by the equation

$$DW^{IV} - \rho_1 h_1 \omega^2 W + \rho g W - \rho\omega^2\Phi = -P(x) \qquad (|x| \leqslant L, \quad y = 0), \tag{1.2}$$

where D is the cylindrical rigidity, ρ_1 and h_1 are the density and thickness of the plate, and ρ is the density of the fluid.

The plate ends are free, which implies that the bending moment and the shear force are equal to zero:

$$W'' = W''' = 0 \qquad (|x| = L).$$

The prime indicates the derivative with respect to x.

Far from the plate, we should satisfy the radiation condition, which means that the surface waves generated by plate oscillations are diverging:

$$\frac{\partial\Phi}{\partial x} \pm i\nu\Phi \to 0 \qquad (x \to \pm\infty). \tag{1.3}$$

After that, we pass to the dimensionless variables (marked by the asterisk)

$$(x_*, y_*, W_*) = \frac{(x, y, W)}{L}, \quad \omega_* = \omega\sqrt{\frac{L}{g}}, \quad \Phi_* = \frac{\Phi}{L^2}, \quad P_* = \frac{P}{\rho g L}.$$

The following dimensionless coefficients are introduced:

$$\delta = \frac{D}{\rho g L^4}, \qquad \chi = \frac{\rho_1 h_1}{\rho L}, \qquad K = \nu L.$$

2. Method of the Solution. In the dimensionless variables (the asterisks are omitted hereinafter), the plate deflection $W(x)$ is sought in the form of the expansion in terms of the normal modes of oscillation of a beam with free ends in vacuum:

$$W(x) = \sum_{n=0}^{\infty} b_n W_n(x). \tag{2.1}$$

In this equation, the complex coefficients b_n are to be determined, and the functions $W_n(x)$ are nontrivial solutions of the following spectral problem:

$$W_n^{IV} = \lambda_n^4 W_n \qquad (|x| \leqslant 1),$$

$$W'_{2k} = W_{2k+1} = 0 \quad (x = 0), \qquad W''_n = W'''_n = 0 \quad (|x| = 1).$$

These solutions have the form

$$\begin{gathered} W_0 = 1/\sqrt{2}, \qquad W_{2k} = D_{2k}[\cos(\lambda_{2k}x) + S_{2k}\cosh(\lambda_{2k}x)], \\ W_1 = \sqrt{3/2}\,x, \qquad W_{2k+1} = D_{2k+1}[\sin(\lambda_{2k+1}x) + S_{2k+1}\sinh(\lambda_{2k+1}x)] \qquad (k = 0, 1, 2, \ldots), \end{gathered} \tag{2.2}$$

where $S_n = \cos\lambda_n/\cosh\lambda_n$ and $D_n = 1/\sqrt{1 + (-1)^n S_n^2}$. The eigenvalues of λ_n are found from the equation

$$\tan\lambda_n + (-1)^n \tanh\lambda_n = 0 \quad (n > 1), \qquad \lambda_0 = \lambda_1 = 0.$$

The functions $W_n(x)$ form a complete orthogonal system for which

$$\int_{-1}^{1} W_n(x) W_m(x)\, dx = \delta_{nm},$$

where δ_{nm} is the Kronecker symbol.

Using expansion (2.1), we can seek for the solution for $\Phi(x, y)$ in the form

$$\Phi(x, y) = \sum_{n=0}^{\infty} b_n \Phi_n(x, y). \tag{2.3}$$

We substitute expansions (2.1) and (2.3) into Eq. (1.2), multiply the resultant relation by $W_m(x)$, and integrate the result with respect to x from -1 to 1. Using the properties of the functions $W_m(x)$, we obtain an infinite system of linear equations for determining the coefficients b_m

$$[1 + \delta(\lambda_m)^4 - \chi K] b_m - K \sum_{n=0}^{\infty} b_n \Psi_{nm} = -Y_m, \tag{2.4}$$

where

$$\Psi_{nm} = \int_{-1}^{1} \Phi_n(x, 0) W_m(x)\, dx, \qquad Y_m = \int_{-1}^{1} P(x) W_m(x)\, dx. \tag{2.5}$$

The values of Ψ_{nm} differ from zero only if both subscripts n and m are either odd or even. Therefore, system (2.4) decomposes into two separate systems: for odd and even values of m.

3. Radiation Problem. To determine the coefficients Ψ_{nm} in (2.5), we have to find the functions $\Phi_n(x, 0)$, which corresponds to solving the so-called radiation problem in terminology of the hydrodynamic theory of ship motion. This problem was previously considered in [3–6] for the first two modes of plate oscillation ($n = 0, 1$), which are further called rigid-body modes. These modes correspond to vertical and rotational oscillations of the rigid plate floating on the fluid surface.

In the formulation considered, the radiation problem has the form

$$\Delta \Phi_n = 0 \qquad (|x| \leqslant \infty, \quad y < 0);$$

$$\frac{\partial \Phi_n}{\partial y} = W_n(x) \qquad (|x| \leqslant 1, \quad y = 0); \tag{3.1}$$

$$\frac{\partial \Phi_n}{\partial y} = K \Phi_n \qquad (|x| > 1, \quad y = 0) \tag{3.2}$$

with conditions in the far field similar to (1.1) and (1.3).

Far from the plate, according to the radiation condition (1.3), we have

$$\Phi_n(x, y) \to H_n^{\pm} \exp\,[K(y \mp ix)] \qquad (x \to \pm\infty),$$

where $H_n^{\pm}$ are the amplitudes of radiation potentials, and by virtue of the assumption on the symmetry properties of the function $W_n(x)$, we obtain

$$H_{2k}^{-} = H_{2k}^{+}, \qquad H_{2k+1}^{-} = -H_{2k+1}^{+} \qquad (k = 0, 1, 2, \ldots). \tag{3.3}$$

Here, we use the method described in [4] to solve the radiation problem. We introduce auxiliary functions $F_n(x, y)$ determined by the relation

$$\frac{\partial F_n}{\partial y} = \frac{\partial \Phi_n}{\partial y} - K \Phi_n.$$

The functions $F_n(x, y)$ satisfy the Laplace equation in the region occupied by the fluid and the boundary conditions

$$\frac{\partial F_n}{\partial y} + K F_n = U_n(x) \quad (|x| \leqslant 1,\ y = 0), \qquad \frac{\partial F_n}{\partial y} = 0 \quad (|x| > 1,\ y = 0),$$

where

$$U_n(x) = W_n(x) + K^2 \int_0^x \int_0^{\xi} W_n(\eta)\, d\eta\, d\xi + K(c_n + d_n x). \tag{3.4}$$

The constants c_n and d_n are to be determined.

We introduce the functions $f_n(x)$ defined on the segment $|x| \leqslant 1$ by the relation

$$f_n(x) = \frac{\partial F_n}{\partial y}\bigg|_{y=0}.$$

The functions $f_n(x)$ determine the hydrodynamic pressure acting on the plate. The solution for $f_n(x)$ can be obtained from the integral equation

$$f_n(x) - \frac{K}{\pi} \int\limits_{-1}^{1} f_n(\xi) \ln |x - \xi| \, d\xi = U_n(x) \qquad (|x| \leqslant 1). \tag{3.5}$$

According to (3.4), the functions $f_n(x)$ depend linearly on the constants c_n and d_n found from the following equations:

$$c_n = \int\limits_0^1 [f_n(x) + f_n(-x)] G(x) \, dx; \tag{3.6}$$

$$d_n = \int\limits_0^1 [f_n(-x) - f_n(x)] G'(x) \, dx. \tag{3.7}$$

Here, the function $G(x)$ can be conveniently represented as [7]

$$G(x) = [\cos(Kx) \operatorname{Ci}(Kx) + \sin(Kx) \operatorname{Si}(Kx) - \ln x]/\pi - \sin(Kx)/2 - i \cos(Kx),$$

where $\operatorname{Si}(\,\cdot\,)$ and $\operatorname{Ci}(\,\cdot\,)$ are the integral sine and cosine, respectively.

We can easily show that $c_{2k+1} = d_{2k} = 0$ $(k = 0, 1, 2, \ldots)$, because the functions $f_n(x)$ possess the same properties of symmetry as the functions $W_n(x)$.

After solving the integral equation (3.5) and determining the complex constants c_n and d_n, the functions $f_n(x)$ are found from relations (3.6) and (3.7).

The unknown quantities Ψ_{nm} are determined from the relation

$$\Psi_{nm} = \frac{2}{K} \int\limits_0^1 [W_n(x) - f_n(x)] W_m(x) \, dx = \frac{2}{K} \Big[\delta_{nm} - \int\limits_0^1 f_n(x) W_m(x) \, dx\Big]. \tag{3.8}$$

The amplitudes of the radiation potentials in the far field are

$$H_{2k}^+ = 2i \int\limits_0^1 f_{2k}(x) \cos(Kx) \, dx,$$

$$H_{2k+1}^+ = -2 \int\limits_0^1 f_{2k+1}(x) \sin(Kx) \, dx \qquad (k = 0, 1, 2, \ldots). \tag{3.9}$$

In the general case, the quantities Ψ_{nm} are complex; by analogy with the hydrodynamic theory of ship motion, they can be presented as

$$\Psi_{nm} = A_{nm} + i B_{nm}, \tag{3.10}$$

where the real values of A_{nm} and B_{nm} are equivalent to the added mass and damping coefficients.

Some useful properties of these coefficients are known:

1) the matrix Ψ_{nm} is symmetric, i.e., $\Psi_{nm} = \Psi_{mn}$;

2) based on the law of conservation of energy, the damping coefficients are expressed via the amplitudes of potentials in the far field with allowance for Eq. (3.3):

$$B_{nm} = H_m^+ \bar{H}_n^+ \tag{3.11}$$

(the bar indicates complex conjugation);

3) the diagonal damping coefficients are always positive:

$$B_{nn} = |H_n^+|^2.$$

The values of Ψ_{nm} and H_n^+ depend on the frequency parameter K. It seems of interest to study their behavior for $K \to 0$ and $K \to \infty$.

4. Limiting Cases of Low and High Oscillation Frequencies. As $K \to 0$, the free surface is equivalent to the rigid wall, according to the boundary condition (3.2). It is known (see, e.g., [3, 6]) that the limiting value of B_{00} in the accepted notation is

$$\lim_{K \to 0} B_{00}(K) = 2.$$

It is of interest to note that this limiting value of the damping coefficient is valid for all two-dimensional contours floating on the free surface and engaged into vertical oscillations under the condition that the length of the line of intersection of the free surface equals the width of the plate considered [8].

The corresponding added mass coefficient has the logarithmic singularity

$$A_{00}(K) \to 2[3/2 - \gamma - \ln(2K)]/\pi \qquad (K \to 0),$$

where $\gamma = 0.577\,21\ldots$ is the Euler constant.

For all other components of the matrix of the damping coefficients, we have

$$\lim_{K \to 0} B_{nm}(K) = 0 \qquad (n + m > 0),$$

and the limiting values of the corresponding components of the added mass matrix are finite and, according to [6], are determined by the relation

$$A_{nm}(0) = \int_{-1}^{1} W_n(x) \int_{-1}^{1} W_m(\xi) S(x - \xi)\, d\xi\, dx, \tag{4.1}$$

where

$$S(\eta) = \frac{1}{\pi} \int_0^\infty \frac{\cos(k\eta)}{k}\, dk.$$

The value of $A_{11}(0) = 1.5/\pi$ is calculated explicitly.

For a high frequency of oscillations ($K \to \infty$), the gravity can be neglected in the radiation problem, and the boundary condition (3.2) acquires the following form for a weightless fluid:

$$\Phi_n = 0 \qquad (|x| > 1, \quad y = 0). \tag{4.2}$$

The behavior of surface waves generated by high-frequency oscillations of the plate was considered in detail in [5]. It was shown that the amplitudes of radiation potentials in the far field are

$$H_n^\pm = i\sqrt{\pi/K}\, U_n^\pm \exp(-iK - i\pi/8) + O(K^{-1}) \qquad (K \to \infty), \tag{4.3}$$

where

$$U_n^\pm = \frac{1}{\pi} \int_{-1}^{1} W_n(x) \Big(\frac{1+x}{1-x}\Big)^{\pm 1/2} dx.$$

Using Eqs. (3.11) and (4.3), we obtain the asymptotic expression for the damping coefficients

$$B_{nm} = \frac{\pi}{K} U_n^+ U_m^+ + O(K^{-3/2}) \qquad (K \to \infty). \tag{4.4}$$

The values of U_n^+ have the form

$$U_0^+ = 1/\sqrt{2}, \qquad U_n^+ = D_n[J_0(\lambda_n) + S_n I_0(\lambda_n)] \qquad (n \geqslant 2)$$

for even modes and

$$U_1^+ = \sqrt{3}/(2\sqrt{2}), \qquad U_n^+ = D_n[J_1(\lambda_n) + S_n I_1(\lambda_n)] \qquad (n \geqslant 3)$$

for odd modes [$J_m(\cdot)$ and $I_m(\cdot)$ are the ordinary and modified Bessel functions of the first kind of order m].

The solution for the radiation potential on the free surface of a weightless fluid is also given in [5]:

$$\Phi_n(x,0) = -\frac{1}{\pi}\int_{-1}^{1} W_n(\xi)\ln\frac{|\sqrt{(1+\xi)/(1-\xi)} - \sqrt{(1+x)/(1-x)}\,|}{\sqrt{(1+\xi)/(1-\xi)} + \sqrt{(1+x)/(1-x)}}\,d\xi.$$

Nevertheless, it is more convenient to use the solution of the radiation problem for a weightless fluid presented in [9, 10], which allows obtaining explicit expressions for all components of the added mass matrix.

According to [11], in the solution of the problem for the Laplace equation in the lower half-plane with the coupled boundary conditions (3.1), (4.2), the horizontal component of fluid velocity on the plate has the form

$$\frac{\partial\Phi_n}{\partial x} = \frac{1}{\pi\sqrt{1-x^2}}\,\text{v.p.}\int_{-1}^{1}\frac{\sqrt{1-\xi^2}}{\xi - x}\,W_n(\xi)\,d\xi \qquad (|x| \leqslant 1, \quad y = 0),$$

where "v.p." indicates the integral in the sense of its principal value.

The behavior of the radiation potential under the plate is calculated as

$$\Phi_n(x,0) = \int_{-1}^{x}\frac{\partial\Phi_n}{\partial x}(\xi,0)\,d\xi,$$

because the values of the potential at the plate ends are equal to zero.

For rigid-body modes of plate oscillations, these solutions are well known (see, e.g., [5, 6]):

$$\Phi_0(x,0) = \sqrt{(1-x^2)/2}, \qquad \Phi_1(x,0) = (x/2)\sqrt{3(1-x^2)/2} \qquad (|x| \leqslant 1) \tag{4.5}$$

and the corresponding values of the added masses are $A_{00} = \pi/4$ and $A_{11} = 3\pi/32$.

The expressions for the radiation potential under the plate have the form

$$\Phi_n(x,0) = D_n\Big\{\sqrt{1-x^2}(J_0^n + S_n I_0^n) + \sum_{k=1}^{\infty}\Big[(-1)^k J_{2k}^n + S_n I_{2k}^n\Big]\Big[\frac{\sin(2k+1)\theta}{2k+1} - \frac{\sin(2k-1)\theta}{2k-1}\Big]\Big\} \tag{4.6}$$

for even elastic modes ($n \geqslant 2$) and the form

$$\Phi_n(x,0) = D_n\Big\{x\sqrt{1-x^2}(J_1^n + S_n I_1^n) + \sum_{k=1}^{\infty}\Big[(-1)^k J_{2k+1}^n + S_n I_{2k+1}^n\Big]\Big[\frac{\sin 2(k+1)\theta}{2(k+1)} - \frac{\sin 2k\theta}{2k}\Big]\Big\} \tag{4.7}$$

for odd elastic modes ($n \geqslant 3$). Here $\theta = \arccos x$, $J_m^n = J_m(\lambda_n)$, and $I_m^n = I_m(\lambda_n)$.

Using solutions (4.5)–(4.7), we can determine all components of the added mass matrix: for even modes,

$$\begin{aligned} A_{0n} &= \pi D_n(J_1^n + S_n I_1^n)/(\lambda_n\sqrt{2}\,), \\ A_{nm} &= \pi D_n D_m\{Q_1^{nm} + Q_2^{nm} \\ &+ [S_n(\lambda_n J_0^m I_1^n + \lambda_m J_1^m I_0^n) + S_m(\lambda_n J_1^n I_0^m + \lambda_m J_0^n I_1^m)]/(\lambda_n^2 + \lambda_m^2)\} \quad (n, m \geqslant 2); \end{aligned} \tag{4.8}$$

for odd modes,

$$\begin{aligned} A_{1n} &= \sqrt{3/2}\,\pi D_n(J_2^n + S_n I_2^n)/(2\lambda_n), \\ A_{nm} &= \pi D_n D_m\{Q_3^{nm} - Q_4^{nm} \\ &+ [S_n(\lambda_n J_1^m I_0^n - \lambda_m J_0^m I_1^n) + S_m(\lambda_m J_1^n I_0^m - \lambda_n J_0^n I_1^m)]/(\lambda_n^2 + \lambda_m^2)\} \quad (n, m \geqslant 3). \end{aligned} \tag{4.9}$$

Here, for $n \neq m$,

$$Q_j^{nm} = Z_j^{nm}/(\lambda_n^2 - \lambda_m^2) \qquad (j = \overline{1,4}),$$

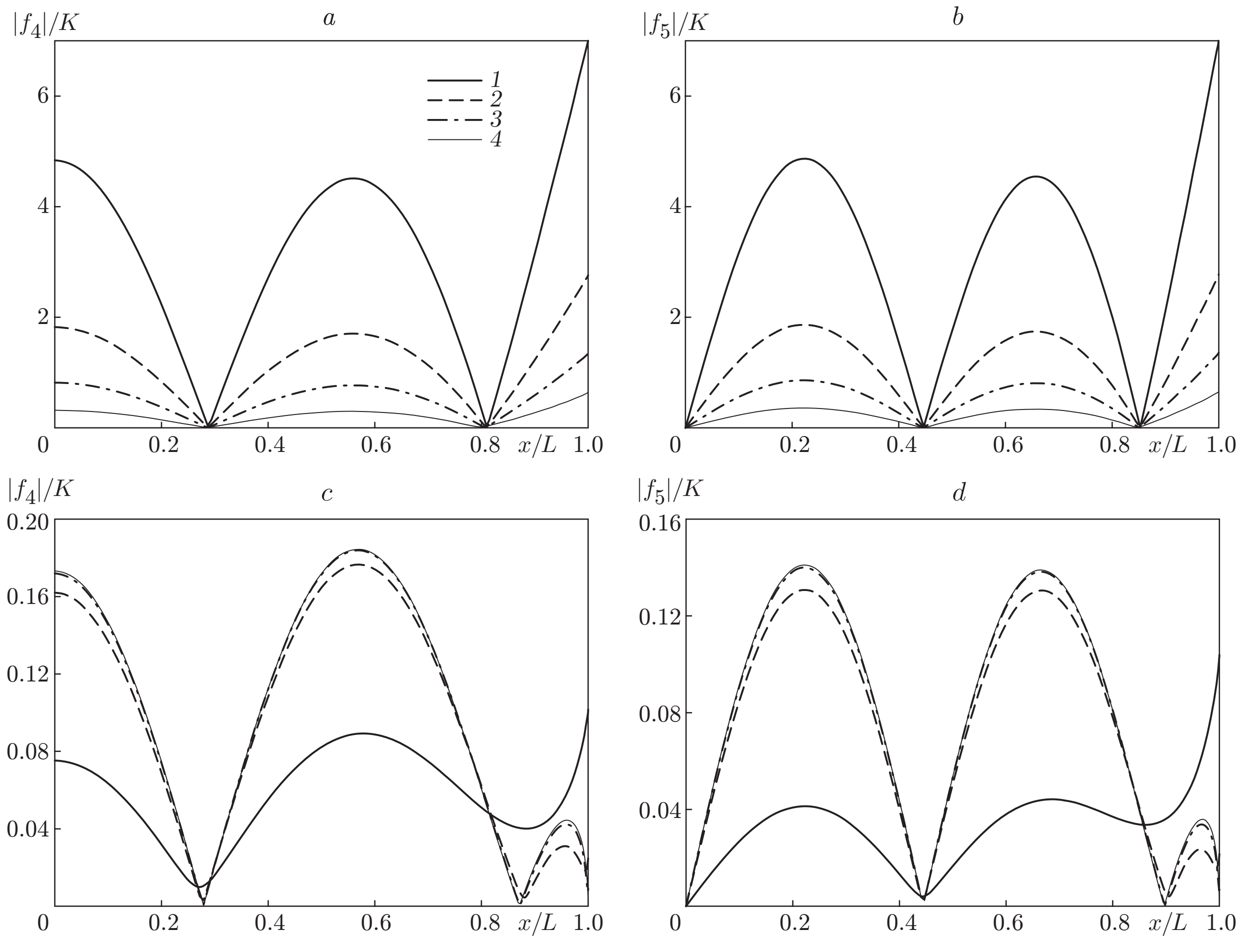

Fig. 1. Behavior of the functions $|f_n(x)|/K$ along the plate for different values of the frequency parameter K: (a) $n = 4$ and $K = 0.2$ (1), 0.5 (2), 1 (3), and 2 (4); (b) $n = 5$ and $K = 0.2$ (1), 0.5 (2), 1 (3), and 2 (4); (c) $n = 4$ and $K = 10$ (1), 100 (2), 1000 (3), and ∞ (4); (d) $n = 5$ and $K = 10$ (1), 100 (2), 1000 (3), and ∞ (4).

$$Z_1^{nm} = \lambda_n P_{mn} - \lambda_m P_{nm}, \qquad Z_2^{nm} = \lambda_n T_{mn} - \lambda_m T_{nm}, \qquad Z_3^{nm} = \lambda_m P_{mn} - \lambda_n P_{nm},$$

$$Z_4^{nm} = \lambda_m T_{mn} - \lambda_n T_{nm}, \qquad P_{nm} = J_0^n J_1^m, \qquad T_{nm} = I_0^n I_1^m,$$

and for $n = m$,

$$Q_1^{nn} = [(J_0^n)^2 + (J_1^n)^2]/2, \qquad Q_2^{nn} = [(I_0^n)^2 - (I_1^n)^2]/2,$$

$$Q_3^{nn} = [(J_1^n)^2 - J_0^n J_2^n)]/2, \qquad Q_4^{nn} = [I_0^n I_2^n - (I_1^n)^2]/2.$$

5. Numerical Calculations. In the numerical solution of the integral equation (3.5), the segment $-1 \leqslant x \leqslant 1$ is divided into $2N$ elements with a uniform step along the variable $\theta = \arccos x$. An additional node is introduced at the mid-point inside each element, and three-point quadratic shape functions are used. With allowance for the properties of evenness for the functions $f_n(x)$, the problem of their determination reduces to solving a system of linear equations of the order $2N + 1$ (or $2N$) for even (or odd) numbers n. In the numerical method used, integration in (3.6)–(3.9) is performed analytically.

For rigid-body modes, the results obtained by the proposed method are compared with tabular values of A_{00}, B_{00}, A_{11}, and B_{11} given in [6] for $0.01 \leqslant K \leqslant 20$. For $N = 100$, the error was within 1%. All the results presented below were obtained for $N = 100$; a further increase in N had practically no effect on the results.

In the present paper, we describe only some examples of solutions obtained for elastic modes with numbers $0 \leqslant n \leqslant 5$. The dependences $|f_n(x)|/K$ for different values of the parameter K for $n = 4$ and 5 are plotted in Fig. 1.

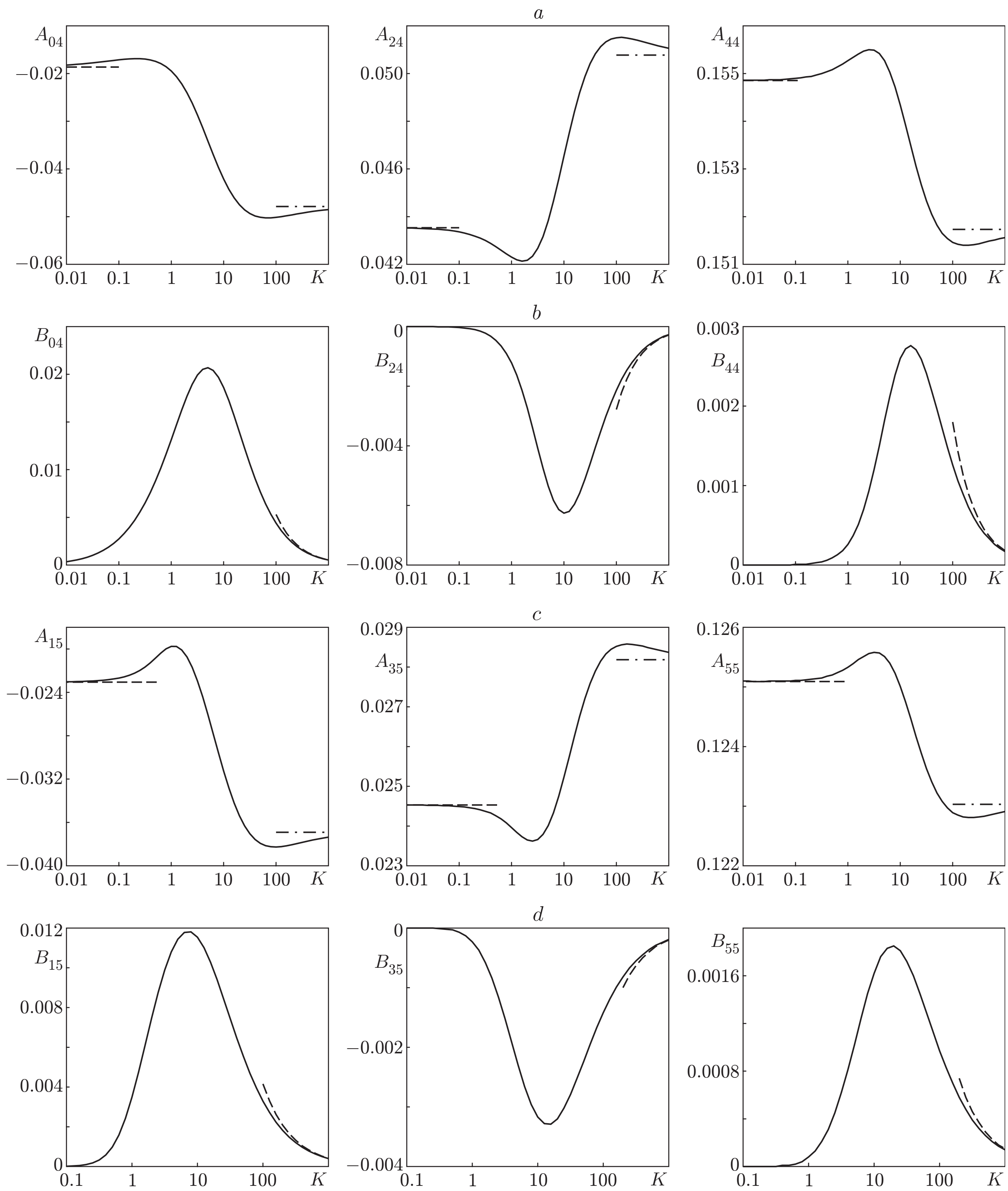

Fig. 2. Coefficients of hydrodynamic loading versus the frequency parameter K: (a, c) added mass coefficients (solid curves) and their limiting values for $K \to 0$ (dashed curves) and $K \to \infty$ (dot-and-dashed curves); (b, d) damping coefficients (solid curves) and their asymptotic dependences for $K \to \infty$ (dashed curves).

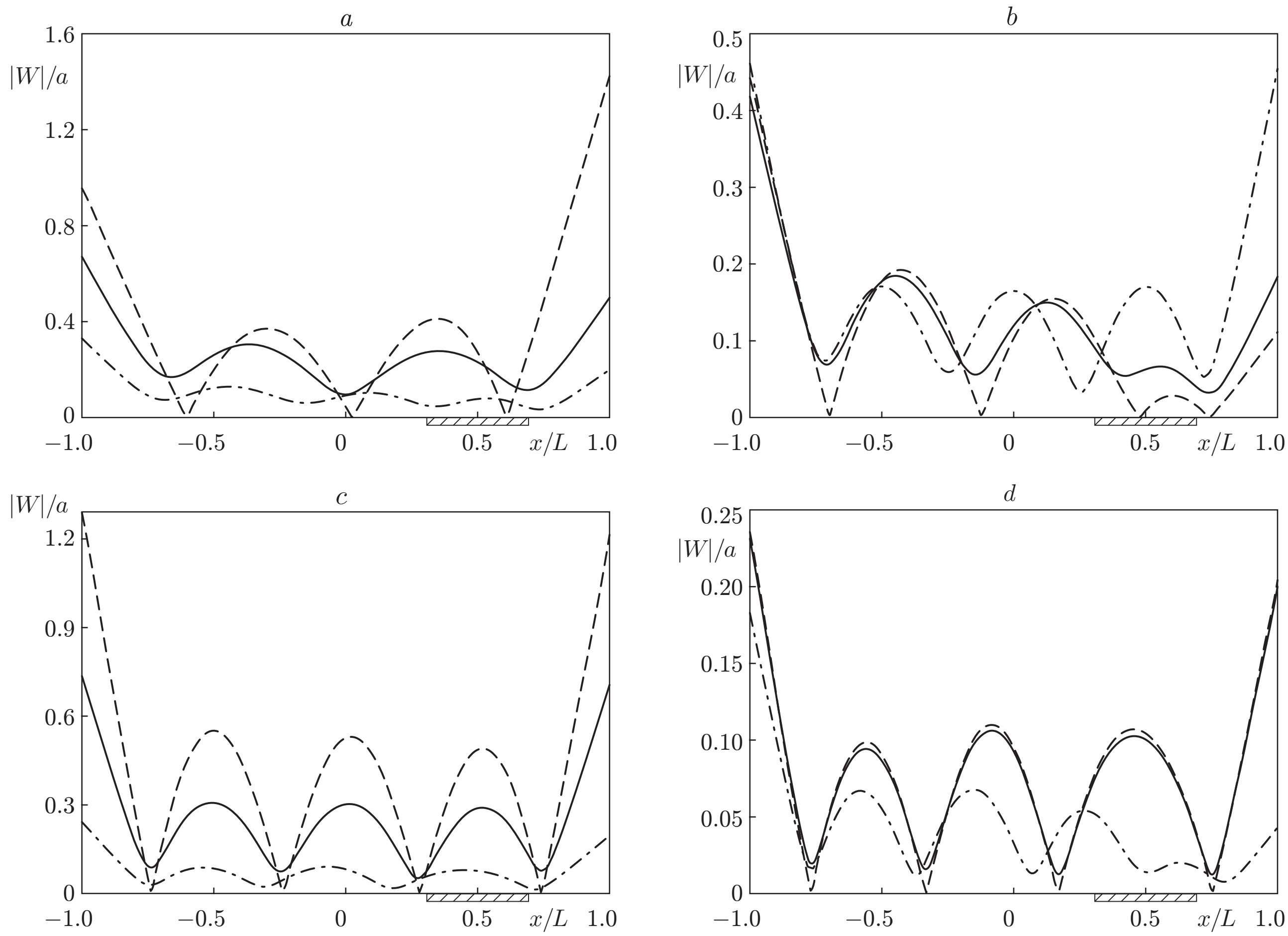

Fig. 3. Amplitudes of normal deflections $|W|/a$ for heavy (solid curves) and weightless (dashed curves) fluids of infinite depth and the results of [1] for a finite-depth heavy fluid (dot-and-dashed curves): $\omega\sqrt{L/g} = 3.162$ (a), 4.111 (b), 5.060 (c), and 6.325 (d); the hatched band shows the region where the pressure is applied.

For $K = \infty$, solutions (4.8) and (4.9) were used for even and odd modes, respectively. It should be noted that the distribution of the functions $f_n(x)$ along the plate for low values of K qualitatively coincides with the behavior of the normal modes $W_n(x)$ described by relations (2.2). With increasing frequency, however, the amplitude of plate deflection at its ends decreases and tends to zero as $K \to \infty$.

The dependences of some elements of the hydrodynamic loading matrix on the parameter K are shown in Fig. 2 for even and odd modes. The horizontal axis is plotted in the logarithmic scale because of the large range of variation of the parameter K. A comparison of the damping coefficients determined by Eq. (3.10) and by the energy relation (3.11) reveals their good agreement. We used the limiting values of the added mass coefficients determined by Eq. (4.1) for $K \to 0$ and by Eqs. (4.8) and (4.9) for $K \to \infty$. The latter limiting dependences for hydrodynamic loading can be used only for very high values of frequency, as least for $K > 10^3$.

We also note that the positions of extrema in the dependences $A_{nm}(K)$ and $B_{nm}(K)$ are shifted toward higher frequencies with increasing numbers n and m. This means that the higher elastic modes are excited only under external actions of sufficiently high frequencies.

The influence of periodic loading on an elastic plate was considered for the following distribution of external pressure in dimensional variables:

$$P(x) = a\rho g[1 - (x - x_0)^2/s^2] \quad (|x - x_0| \leqslant s), \qquad P(x) = 0 \quad (|x - x_0| > s)$$

(a is a factor with a dimension of length). We assumed that $|x_0| + s < L$.

The values of the initial parameters were the same as in [1]: $D = 1.093 \cdot 10^3$ kg $\cdot$ m^2/sec^2, $\rho = 10^3$ kg/m^3, $\rho_1 h_1 = 12.5$ kg/m^2, $L = 2.5$ m, and $s = 0.5$ m.

The system of linear equations (2.4) was solved by the reduction method, an infinite series being replaced by a finite sum with the number of terms M. The calculations described involved $M = 20$ modes.

Figure 3 shows the amplitudes of normal deflections of the plate W/a for $x_0/L = 0.5$ and four different values of the dimensionless frequency $\omega\sqrt{L/g}$. The solution for a weightless fluid involved only the added mass coefficients determined by Eqs. (4.8) and (4.9). The dot-and-dashed curves show the amplitudes of deflections under the action of the same load for a plate floating on the surface of a fluid with a finite depth equal to 0.25 m. Comparing the solutions for a heavy fluid of finite and infinite depths, we should note their substantial discrepancy. Normally, the amplitudes of plate deflections in the case of an infinitely deep fluid exceed the corresponding values for a finite-depth fluid.

The solution for an infinitely deep weightless fluid yields satisfactory agreement with the solution for a heavy fluid only if the frequency is fairly high (see Fig. 3d); for lower frequencies, the deflection is usually overpredicted.

Note, a typical feature for all solutions is that the deflection at the plate ends is much higher than the deflection in the middle part of the plate.

Conclusions. The results presented show that the depth of the fluid has a significant effect on the behavior of an elastic plate subjected to external periodic loading. The detailed study of the dependence of the elements of the hydrodynamic loading matrix on frequency can be used to solve the problem of the action of a generic unsteady load on a floating elastic plate.

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